# Integration by Tables (page 1 of 2)

As you may have already noticed, the back pages of the textbook contain 120 formulas for integration. Using tables isn't always as easy as it may seem. You will often need to use u-substitution or a second or third formula.

Table Categories: Basic forms, $\sqrt{a^2+u^2}$, $\sqrt{a^2-u^2}$, $\sqrt{u^2-a^2}$, $(a+bu)$, Trig. Fcns., Inverse Trig. Fcns., Exponents and Logarithms, Hyperbolic Fcns., and $\sqrt{2au-u^2}$.

→ "a" is constant, "u" is function.

Ex: $\int \frac{x}{\sqrt{1+x}}\,dx$ Let's try "a+bu".

Formula 55: $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$

For our example: $u = x$ $\quad a = 1$

$du = dx$ $\quad b = 1$

$\therefore \int \frac{x}{\sqrt{1+x}}\,dx = \frac{2}{3}(x-2)\sqrt{1+x}+C$

Ex: $\int x^2\sqrt{2+9x^2}\,dx$ Once you find a formula, rewrite integral to match.

Formula 22: $\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln\left(u+\sqrt{a^2+u^2}\right)+C$

$u = 3x$ $\quad a = \sqrt{2}$

$du = 3dx$ $\quad a^2 = 2$

$\frac{1}{3}\cdot\frac{1}{9}\int (3x)^2\sqrt{(\sqrt{2})^2+(3x)^2}\,\underbrace{3dx}_{du}$ ← Pay attention to all constants. Be careful with dx to du.

$= \frac{1}{27}\int u^2\sqrt{(\sqrt{2})^2+u^2}\,du$

$= \frac{1}{27}\left(\frac{3x}{8}(2+2\cdot 9x^2)\sqrt{2+9x^2} - \frac{4}{8}\ln\left(3x+\sqrt{2+9x^2}\right)\right)+C$

$= \frac{1}{216}\left(3x(2+18x^2)\sqrt{2+9x^2} - 4\ln\left(3x+\sqrt{2+9x^2}\right)\right)+C$

Ex: $\int x^3\sin x\,dx$ ← We can do this one by parts.

$\int x^3 \sin x\,dx$ $\quad u = x^3 \quad du = 3x^2\,dx \quad dv = \sin x\,dx \quad v = -\cos x$

Integration by Parts THREE times.

$= -x^3\cos x + \int 3x^2 \cos x\,dx$ $\quad u = x^2 \quad du = 2x\,dx \quad dv = \cos x\,dx \quad v = \sin x$

$= -x^3\cos x + 3\left(x^2\sin x - \int 2x\sin x\,dx\right)$ $\quad u = x \quad du = dx \quad dv = \sin x\,dx \quad v = -\cos x$

$= -x^3\cos x + 3x^2\sin x - 6\left(-x\cos x + \int \cos x\,dx\right)$

$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

$= (6x - x^3)\cos x + (3x^2 - 6)\sin x + C$

## Reduction Formulas can simplify this.

Reduction formulas include a new integral with a smaller degree. (Don't forget to account for $dx \to du$ conversion, if needed.)

Formula 82: $\int u\sin u\,du = \sin u - u\cos u + C$

Formula 83: $\int u\cos u\,du = \cos u + u\sin u + C$

Formula 84: $\int u^n \sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

Formula 85: $\int u^n \cos u\,du = u^n \sin u - n\int u^{n-1}\sin u\,du$

Examples of Reduction Formulas

Ex: $\int x^3 \sin x\,dx$

84 $\quad u = x \quad n = 3 \quad du = dx$

$= -x^3\cos x + 3\int x^2\cos x\,dx$

85 $\quad u = x \quad n = 2 \quad du = dx$

$= -x^3\cos x + 3\left(x^2\sin x - 2\int x\sin x\,dx\right)$

82 $\quad u = x \quad du = dx$

$= -x^3\cos x + 3x^2\sin x - 6(\sin x - x\cos x) + C$

↑ same as above

On a test or a quiz, I may give you an integral and a short list of formulas. This would test if you can apply the formulas correctly.

Note: Understanding how to apply the formulas is an important skill. However, throughout the year you should do all integrals by hand. Refer to the tables only when stuck.