Quiz 11.1 - 11.3 - *Solve without comparison tests.* ______ Name: Key

Form A (2 pts. per problem) 20 pts. Per.: ______

1) Write the first 5 terms of the following sequence beginning with $n = 1$.
Converges or diverges? Converges
If converges, find limit.

$$\left\{\frac{2n-1}{3n+1}\right\} \quad \boxed{\frac{1}{4}, \frac{3}{7}, \frac{5}{10}, \frac{7}{13}, \frac{9}{16}}$$

$$\lim_{n\to\infty} \frac{2n-1}{3n+1} = \frac{2}{3} \qquad \text{Limit} = \boxed{\frac{2}{3}}$$

2) For $\sum_{n=1}^{\infty} \frac{1}{n}$ write both S_4 and S_5, but you don't need to calculate them.

$$S_4 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \boxed{\frac{25}{12}}$$

$$S_5 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \boxed{\frac{137}{60}}$$

For #3 - 8, state (a) converges or diverges and (b) how you know (e.g., test or series name). Do NOT find sums.

3) $\sum_{n=0}^{\infty} \frac{n-9}{n}$

a) Diverges

b) n^{th} term test

4) $\sum_{n=0}^{\infty} \frac{1}{4^n}$

a) Converges

b) Geometric $r = \frac{1}{4} < 1$

5) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

a) Diverges

b) p-series $p = \frac{1}{2} < 1$

6) $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

a) Converges

b) Telescoping

Sum = 1

7) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

a) Diverges

b) Integral Test

8) $\sum_{n=1}^{\infty} 4ne^{-n}$

a) Converges

b) Integral Test

Note: integral converges to $\frac{8}{e}$ and needs L'Hospital's Rule

For #9 and 10, show that the series converges and find the sum of the series.

9) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

Telescoping

$$1 = A(n+2) + Bn$$

$$1 = An + 2A + Bn$$

$$A + B = 0, \; 2A = 1 \Rightarrow A = \frac{1}{2}, \; B = -\frac{1}{2}$$

$$\sum \frac{1}{2n} - \frac{1}{2(n+2)} = \frac{1}{2}\sum \frac{1}{n} - \frac{1}{n+2}$$

$$S_n = \frac{1}{2}\left(\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \ldots + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+2}\right)\right)$$

$$S_n = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$\lim_{n\to\infty} S_n = \frac{1}{2}\left(1 + \frac{1}{2} + 0 - 0\right) = \boxed{3/4}$$

$\sum a_n$ converges to the sum 3/4

10) $\sum_{n=1}^{\infty} 3(-0.6)^n$

Geometric $|r| = 0.6 < 1 \Rightarrow$ converges

$$\sum_{n=1}^{\infty} 3(-0.6)^n = \sum_{n=1}^{\infty} 3(-0.6)(-0.6)^{n-1}$$

$$a = 3(-0.6) \qquad r = -0.6$$

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} = \frac{3(-0.6)}{1 - {-0.6}}$$

$$= \frac{-1.8}{1.6} = -\frac{18}{16} = \boxed{-\frac{9}{8}}$$

common error: 15/8 wrong since did not adjust for $n=1$ starting value.