Quiz 11.1 - 11.7 ______ Name: Key

Form A (2 pts. per problem #1-8) 20 pts. Per.: ______

1) Does this sequence converge or diverge?

Diverges If it converges, find limit.

$$\left\{\frac{6n^2-1}{7n+1}\right\}$$

$\lim_{n\to\infty} \frac{6n^2-1}{7n+1} = \infty$ DNE

2) Does this sequence converge or diverge?

Converges If it converges, find limit.

$$\left\{\frac{5n^3-n^2}{3n^3+1}\right\}$$

$\lim_{n\to\infty} \frac{5n^3-n^2}{3n^3+1} = \boxed{\frac{5}{3}}$

For #3 - 8, state (a) converges or diverges and (b) how you know (e.g., test or series name). Do NOT find sums.

3) $$\sum_{n=1}^{\infty}\left(\frac{4n-1}{3n+5}\right)^n$$

a) Diverges

b) Root Test

$\rho = \frac{4}{3} > 1$

4) $$\sum_{n=1}^{\infty} ne^{-n^2}$$

a) Converges

b) Integral Test

$\lim_{b\to\infty}\int_1^b xe^{-x^2}\,dx = \frac{1}{2e}$

5) $$\sum_{n=1}^{\infty}\sqrt{\frac{1}{n^5}}$$

a) Converges

b) P-Series

$p = \frac{5}{2} > 1$

6) $$\sum_{n=1}^{\infty}(-1)^n\left(\frac{8n-n^2}{3n}\right)$$

a) Diverges

b) n^{th} term test

Alternating Series Test Fails

$\lim_{n\to\infty}\frac{8n-n^2}{3n} = -\infty \neq 0$

7) $$\sum_{n=1}^{\infty}\frac{1}{3^n+2}$$

a) Converges

b) Direct comparison to $\sum \frac{1}{3^n}$ ← geometric converges $|r| = \frac{1}{3} < 1$

$\frac{1}{3^n} \geq \frac{1}{3^n+2}$

8) $$\sum_{n=1}^{\infty}\frac{e^n}{n!}$$

a) Converges

b) Ratio Test

$\rho = 0 < 1$

9) Does the following series converge or diverge? If it converges, determine if it converges absolutely or conditionally? *Show all work and name tests used*. (4 pts.)

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n(n+1)}}$$

converges conditionally

Alternating Series Test

① $b_{n+1} \leq b_n$

$\frac{1}{\sqrt{(n+1)(n+1+1)}} \leq \frac{1}{\sqrt{n(n+1)}}$

$\sqrt{n^2+3n+2} \geq \sqrt{n^2+n}$

$\cancel{n^2}+3n+2 \geq \cancel{n^2}+n$

$2n \geq -2$

$n \geq -1$

True $\forall n$ ✓

② $\lim_{n\to\infty} b_n = 0$?

$\lim_{n\to\infty}\frac{1}{\sqrt{n(n+1)}} = 0$ ✓

$\therefore \sum a_n$ converges by the Alternating Series Test

check $\sum |a_n|$

$\sum\frac{1}{\sqrt{n(n+1)}} = \sum\frac{1}{\sqrt{n^2+n}}$ limit comparison to $\sum\frac{1}{n}$ ← Diverges by Integral Test

$\lim_{n\to\infty}\frac{1}{\sqrt{n^2+n}}\cdot\frac{n}{1}$

$= \lim_{n\to\infty}\frac{n}{\sqrt{n^2+n}}\cdot\frac{1/n}{1/\sqrt{n^2}}$

$= \lim_{n\to\infty}\frac{1}{\sqrt{1+1/n}} = 1$ ← positive & finite

$\therefore$ series do same thing

$\Rightarrow \sum\frac{1}{\sqrt{n(n+1)}}$ also diverges

$\therefore \sum(-1)^n\frac{1}{\sqrt{n(n+1)}}$ converges conditionally