Quiz 9.1 - 9.5 ______ Name: Key

Show *all* work for full credit. No calculators. 12 pts. Per.: ______

Note: $e \approx 2.71828$ and $1/e \approx 0.367879$

1) a) Solve the following differential equation using *separation of variables*.

$y' + 2xy = 2x$ (3 pts.)

b) Check your answer using the original differential equation. (1 pt.)

Same

$y' + 2xy = 2x$

$y' = 2x - 2xy$

$\frac{dy}{dx} = 2x(1-y)$

$\frac{1}{1-y}\,dy = 2x\,dx$

$-\ln|1-y| = x^2 + C$

$\ln|1-y| = Ce^{-x^2}$

a) $\boxed{y = 1 - Ce^{-x^2}}$

Same

Don't cross the = with this.

b) $y' + 2xy = 2x$

$(0 - C(-2x)e^{-x^2}) + 2x(1 - Ce^{-x^2}) = 2x$

$\cancel{2Cx}e^{-x^2} + 2x - \cancel{2Cx}e^{-x^2} = 2x$

$2x = 2x$

LHS = RHS ✓

2) a) Solve the following IVP using *an integrating factor*.

$y' + 2xy = 2x, \quad y(0) = 2$ (3 pts.)

b) What is the exact value of $y(1)$? (1 pt.)

$P(x)$

$y' + (2x)y = 2x$

$\int P(x)\,dx = \int 2x\,dx = x^2$

Integrating Factor = e^{x^2}

$y'e^{x^2} + 2xye^{x^2} = 2xe^{x^2}$

$\frac{d}{dx}[ye^{x^2}] = 2xe^{x^2}$

$ye^{x^2} = \int 2xe^{x^2}\,dx$

$ye^{x^2} = e^{x^2} + C$

$y = 1 + Ce^{-x^2}$

$y(0) = 2 \Rightarrow 2 = 1 + Ce^0 \Rightarrow C = 1$

a) $\boxed{y = 1 + e^{-x^2}}$

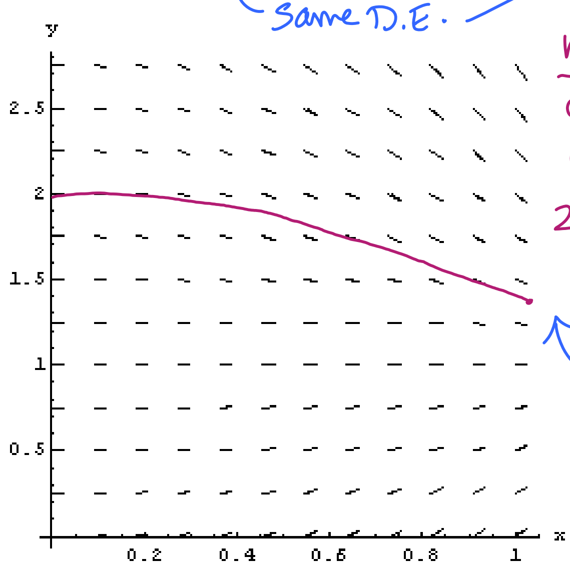
b) $y(1) = 1 + e^{-1} = \boxed{1 + \frac{1}{e}} \approx 1.367879$

3) a) Use *Euler's Method* with a step size of 0.5 to estimate the value of $y(1)$ where y is the solution of the initial-value problem:

$y' = 2x - 2xy, \quad y(0) = 2$. (3 pts.)

b) Use the given direction field to sketch the solution that satisfies the initial condition $y(0) = 2$. (1 pt.)

Same D.E.



n	x	y
0	0	2
1	0.5	2
2	1.0	1.5

Compare

$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$

$y_1 = y_0 + 0.5F(x_0, y_0)$

$y_1 = 2 + 0.5(2(0) - 2(0)(2))$

$= 2 + 0$

$y_2 = 2 + 0.5(2(0.5) - 2(0.5)(2))$

$= 2 + 0.5(1 - 2)$

$= 2 - 0.5$

$= 1.5$

$\boxed{\therefore y(1) \approx 1.5}$