Larson 7.7 L'Hôpital's Rule

Name: _____

Copy exercises and show all work on separate paper.

In Exercises 1–6, evaluate the limit (a) using techniques from Chapters 1 and 3 and (b) using L'Hôpital's Rule.

1. $\lim_{x\to 3}\frac{2(x-3)}{x^2-9}$
2. $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$
3. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$
4. $\lim_{x\to 0}\frac{\sin 4x}{2x}$
5. $\lim_{x\to \infty}\frac{5x^2-3x+1}{3x^2-5}$
6. $\lim_{x\to \infty}\frac{2x+1}{4x^2+x}$

In Exercises 7–26, evaluate the limit, using L'Hôpital's Rule if necessary. (In Exercise 13, n is a positive integer.)

7. $\lim_{x\to 2}\frac{x^2-x-2}{x-2}$
8. $\lim_{x\to -1}\frac{x^2-x-2}{x+1}$
9. $\lim_{x\to 0}\frac{\sqrt{4-x^2}-2}{x}$
10. $\lim_{x\to 2^-}\frac{\sqrt{4-x^2}}{x-2}$
11. $\lim_{x\to 0}\frac{e^x-(1-x)}{x}$
12. $\lim_{x\to 0^+}\frac{e^x-(1+x)}{x^3}$
13. $\lim_{x\to 0^+}\frac{e^x-(1+x)}{x^n}$
14. $\lim_{x\to 1}\frac{\ln x}{x^2-1}$
15. $\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}$
16. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$
17. $\lim_{x\to 0}\frac{\arcsin x}{x}$
18. $\lim_{x\to 1}\frac{\arctan x-(\pi/4)}{x-1}$
19. $\lim_{x\to \infty}\frac{3x^2-2x+1}{2x^2+3}$
20. $\lim_{x\to \infty}\frac{x-1}{x^2+2x+3}$
21. $\lim_{x\to \infty}\frac{x^2+2x+3}{x-1}$
22. $\lim_{x\to \infty}\frac{x^2}{e^x}$
23. $\lim_{x\to \infty}\frac{x}{\sqrt{x^2+1}}$
24. $\lim_{x\to \infty}\frac{\sin x}{x-\pi}$
25. $\lim_{x\to \infty}\frac{\ln x}{x}$
26. $\lim_{x\to \infty}\frac{e^x}{x}$

In Exercises 27–40, describe the type of indeterminate form (if any) that is obtained with direct substitution. Then evaluate the limit, using L'Hôpital's Rule when necessary. (For a geometric approach to Exercise 27, see the article by John H. Mathews in the May, 1992 issue of *The College Mathematics Journal*.)

27. $\lim_{x\to 0^+}(-x\ln x)$
28. $\lim_{x\to 0^+}x^2\cot x$
29. $\lim_{x\to \infty}\left(x\sin\frac{1}{x}\right)$
30. $\lim_{x\to \infty}x\tan\frac{1}{x}$
31. $\lim_{x\to 0^+}x^{1/x}$
32. $\lim_{x\to 0^+}(e^x+x)^{1/x}$
33. $\lim_{x\to \infty}x^{1/x}$
34. $\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^x$
35. $\lim_{x\to 0^+}(1+x)^{1/x}$
36. $\lim_{x\to \infty}(1+x)^{1/x}$
37. $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$
38. $\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)$
39. $\lim_{x\to 1^+}\left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$
40. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$

41. Find differentiable functions f and g that satisfy the specified condition such that $\lim_{x\to 5}f(x)=0$ and $\lim_{x\to 5}g(x)=0$.

    a. $\lim_{x\to 5}\frac{f(x)}{g(x)}=10$ **b.** $\lim_{x\to 5}\frac{f(x)}{g(x)}=0$

    c. $\lim_{x\to 5}\frac{f(x)}{g(x)}=\infty$

    (*Note*: There are many correct answers.)

42. Find differentiable functions f and g such that

    $\lim_{x\to \infty}f(x)=\lim_{x\to \infty}g(x)=\infty$ and $\lim_{x\to \infty}[f(x)-g(x)]=25$.

    (*Note*: There are many correct answers.)