Quiz 12.1 - 12.4 ______ Name: Key

Show all work and circle answers. 15 pts. Per.: _____

No Calculators.

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1) Let **v** have initial point (3, 7) and terminal point (3, -2). Find the *component form* of **v**. (1 pt.)

$\vec{v} = \langle 3-3, -2-7 \rangle$

$\boxed{\vec{v} = \langle 0, -9 \rangle}$

2) Find a unit vector in the direction of **v** = 3**i** - 2**j**. (2 pts.)

$\frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 3, -2 \rangle}{\sqrt{9+4}} = \boxed{\left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle}$

3) Write the standard equation of a sphere that has points (4, -3, 5) and (-6, 1, -1) as endpoints of a diameter. (3 pts.)

center $= \left(\frac{4+-6}{2}, \frac{1+-3}{2}, \frac{-1+5}{2} \right)$

$= (-1, -1, 2)$

diameter $= \sqrt{(4--6)^2 + (1--3)^2 + (-1-5)^2}$

$= \sqrt{100+16+36} = \sqrt{152}$

$r = \frac{1}{2}\sqrt{152} = \frac{\sqrt{152}}{2}$

$r^2 = \frac{(\sqrt{152})^2}{4} = 38$

$\boxed{(x+1)^2 + (y+1)^2 + (z-2)^2 = 38}$

4) Is **u** = 8**i** + 4**j** + 10**k** parallel to **v** = -4**i** - 2**j** + 5**k**? If so, find *c* such that **u** = *c***v**. (1 pt.)

$\boxed{\text{No}}$

5) **u** = <3, 2>, **v** = <1, -1>. Find **u** • **v**. Are **u** and **v** orthogonal? (2 pts.)

$\vec{u} \cdot \vec{v} = 3 \cdot 1 + 2 \cdot (-1) = 3 + -2 = \boxed{1}$

ortnogonal? $\boxed{\text{No}}$

6) **v** = <3, 2>, **w** = <1, -3> Find the projection of **v** onto **w**. (2 pts.)

$\text{Proj}_{\vec{w}} \vec{v} = \left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2} \right) \vec{w} = \left(\frac{3+-6}{(\sqrt{1+9})^2} \right) \langle 1, -3 \rangle$

$= -\frac{3}{10} \langle 1, -3 \rangle$

On a test it will be more clear which projection to find.

intended answer

$\boxed{\left\langle \frac{-3}{10}, \frac{9}{10} \right\rangle}$

Scalar Projection

$\text{comp}_{\vec{w}} \vec{v} = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|} = \frac{-3}{\sqrt{10}}$

7) Calculate the angle **v** = 3**i** - 5**j** + **k** makes with the positive *y*-axis. Leave your answer in "arc-trig" form. (2 pts.)

$\vec{v} = \langle 3, -5, 1 \rangle, \vec{u} = \langle 0, 1, 0 \rangle$

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta \Rightarrow \cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$

$\cos\theta = \frac{(3 \cdot 0 + (-5)(1) + 1 \cdot 0)}{\sqrt{9+25+1} \ \sqrt{0+1+0}}$

$\cos\theta = \frac{-5}{\sqrt{35}} \Rightarrow \theta = \boxed{\arccos\left(\frac{-5}{\sqrt{35}}\right)}$

8) Find ||**u** × **v**|| for **u** = <-1, 2, 2> and **v** = <3, -2, 1>. (2 pts.)

$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 3 & -2 & 1 \end{vmatrix} = (2--4)\hat{i} - (-1-6)\hat{j} + (2-6)\hat{k}$

$= \langle 6, 7, -4 \rangle$

$\|\vec{u} \times \vec{v}\| = \sqrt{36+49+16} = \boxed{\sqrt{101}}$

Make sure cross product is correct. This would be checked, especially $\hat{j}$ term.

Bonus) What does the result in #8 represent? (+1 pt.)

Area of a parallelogram with $\vec{u}$ and $\vec{v}$ as adjacent sides