Chapter 16 Summary

Name: ____________________

16.1 & 16.5 Vector Fields

$\vec{F}(x,y) = P(x,y)\,\hat{\imath} + Q(x,y)\,\hat{\jmath}$

$\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z)\rangle$

$\vec{F}$ is <u>conservative</u> if $\vec{F} = \vec{\nabla} f$ — potential function

2D: $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$

3D: $\overrightarrow{\text{curl}}\,\vec{F} = \vec{0}$

$f = \int P\,dx$
$f = \int Q\,dy$
$f = \int R\,dz$

$$\text{div}\,\vec{F} = \vec{\nabla}\cdot\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\overrightarrow{\text{curl}}\,\vec{F} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

16.2 Line Integrals

<u>Scalar</u>: $\int_C f\,ds$, $ds = \|\vec{r}\,'(t)\|\,dt$

If f is density, this calculates mass.

Arc length: $s = \int_C ds = \int_a^b \|\vec{r}\,'(t)\|\,dt$

<u>Vector</u>: $\int_C \vec{F}\cdot d\vec{r}$, $d\vec{r} = \vec{r}\,'(t)\,dt$

This calculates work.

Also, $\int_C \vec{F}\cdot\vec{T}\,ds = \int_C P\,dx + Q\,dy + R\,dz$

May need to parametrize C.

16.3 Fundamental Theorem of Line Integrals: $\int_C \vec{F}\cdot d\vec{r} = f\big|_{t=b} - f\big|_{t=a}$ where $\vec{F} = \vec{\nabla} f$

16.4 Green's Theorem: $\int_C \vec{F}\cdot d\vec{r} = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$ (2D, counterclockwise orientation)

C \ $\vec{F}$	Conservative	Not Conservative
Closed	$\int_C \vec{F}\cdot d\vec{r} = 0$	Green's Thm. (2D) Stokes' Thm. (3D)
Not Closed	Fundamental Theorem of Line Integrals	Long Way $d\vec{r} = \vec{r}\,'(t)\,dt$

16.6 Parametric Surfaces

$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$

Can eliminate parameter for some surfaces.

Good for surfaces of revolution.

$\vec{r}_u \times \vec{r}_v$ is normal to surface.

$S = \iint_D \|\vec{r}_u \times \vec{r}_v\|\,du\,dv$

16.7 Surface Integrals

<u>Scalar</u>: $\iint_S f\,dS$, $dS = \|\vec{\nabla} G\|\,dA$ or $dS = \|\vec{r}_u \times \vec{r}_v\|\,dA$

If f is density, this calculates mass.

Surface Area: $S = \iint_S dS = \iint_R \|\vec{\nabla} G\|\,dA = \iint_D \|\vec{r}_u \times \vec{r}_v\|\,dA$

<u>Vector</u>: $\iint_S \vec{F}\cdot d\vec{S}$, $d\vec{S} = \vec{\nabla} G\,dA$ or $d\vec{S} = (\vec{r}_u \times \vec{r}_v)\,dA$

This calculates flux. Also, $\iint_S \vec{F}\cdot\vec{N}\,dS$, $d\vec{S} = \vec{N}\,dS$

$x = g(y,z) \Rightarrow dz\,dy$
$y = g(x,z) \Rightarrow dx\,dz$
$z = g(x,y) \Rightarrow dy\,dx$

upward orientation:
$G(x,y,z) = z - g(x,y)$

Positive: upward or outward
Negative is opposite

16.9 Divergence Theorem

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{div}\,\vec{F}\,dV$$

S must be closed

16.8 Stokes' Theorem

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \overrightarrow{\text{curl}}\,\vec{F}\cdot d\vec{S}$$

C must be closed

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