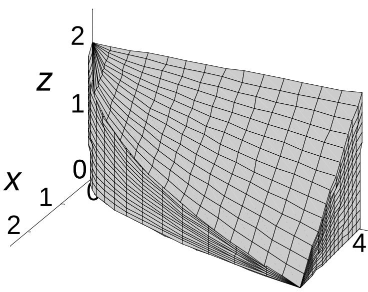
Practicing Limits for Triple Integrals

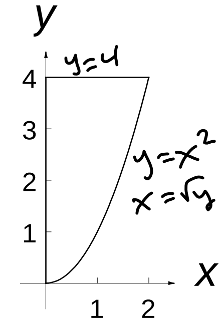
Name: Key

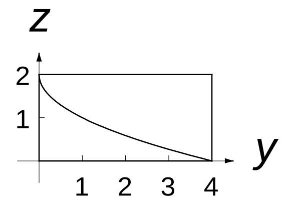
Follow the directions for each problem to practice setting up limits for triple integrals.

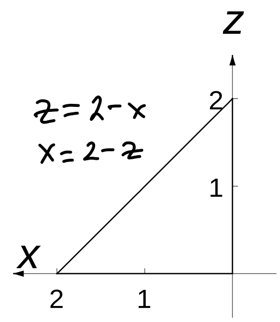
1) Write the integral for *f* over the solid bounded by the given surfaces using four orders of integration.

$$x=0,\quad x=2,\quad y=x^2,\quad y=4,\quad z=0,\quad z=2-x$$









$$\int_0^2\int_{x^2}^4\int_0^{2-x} f(x,y,z)\,dz\,dy\,dx \qquad \int_0^4\int_0^{\sqrt{y}}\int_0^{2-x} f(x,y,z)\,dz\,dx\,dy$$

$$\int_0^2\int_0^{2-x}\int_{x^2}^4 f(x,y,z)\,dy\,dz\,dx \qquad \int_0^2\int_0^{2-z}\int_{x^2}^4 f(x,y,z)\,dy\,dx\,dz$$

If you integrate with respect to *x* first, you need two triple integrals. Why? Try it if you have time.

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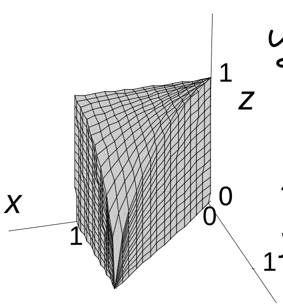
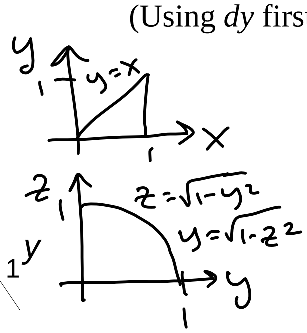
2) Set up two integrals to find the volume of the solid in the first octant bounded by the surfaces $x=y$, $x=1$, and $y^2+z^2=1$. Use *dz* first for one integral and use *dx* first for the other integral.

(Using *dy* first requires two integrals. Try it, if interested.)





$$V=\iiint_E dV \qquad V=\int_0^1\int_0^x\int_0^{\sqrt{1-y^2}} dz\,dy\,dx$$

$$V=\int_0^1\int_y^1\int_0^{\sqrt{1-y^2}} dz\,dx\,dy$$

$$V=\int_0^1\int_0^{\sqrt{1-y^2}}\int_y^1 dx\,dz\,dy=\int_0^1\int_0^{\sqrt{1-z^2}}\int_y^1 dx\,dy\,dz$$

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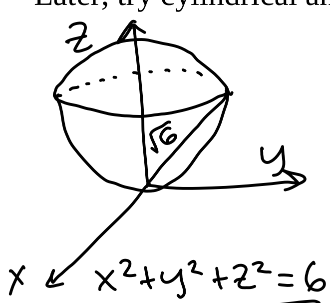
3) Set up a triple integral to find the volume of the solid bounded by the surfaces $x^2+y^2+z^2=6$ and $z=x^2+y^2$. Set this up in rectangular coordinates. How would you set this up using a double integral? Later, try cylindrical and/or spherical coordinates.



$x^2+y^2+z^2=6$

$z=\sqrt{6-x^2-y^2}$

$z=\sqrt{6-r^2}$

$z=x^2+y^2=r^2$

$r^2=\sqrt{6-r^2}$

$r^4=6-r^2$

$r^4+r^2-6=0$

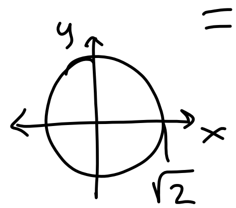
$(r^2+3)(r^2-2)=0$

$r^2=2$

$r=\sqrt{2}$

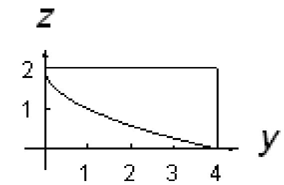
$z=2$

$$V=\iiint_E dV=\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}}\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz\,dy\,dx$$



Practicing Limits for Triple – Additional Answers

① Integrating dx first:



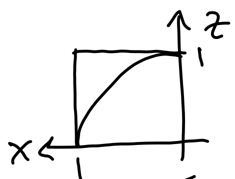
$z = 2-\sqrt{y}$

$y = (2-z)^2$

$$\int_0^4\int_0^{2-\sqrt{y}}\int_0^{\sqrt{y}} f(x,y,z)\,dx\,dz\,dy + \int_0^4\int_{2-\sqrt{y}}^{2}\int_0^{2-z} f(x,y,z)\,dx\,dz\,dy$$

$$\int_0^2\int_0^{(2-z)^2}\int_0^{\sqrt{y}} f(x,y,z)\,dx\,dy\,dz + \int_0^2\int_{(2-z)^2}^{4}\int_0^{2-z} f(x,y,z)\,dx\,dy\,dz$$

② Integrating dy first:



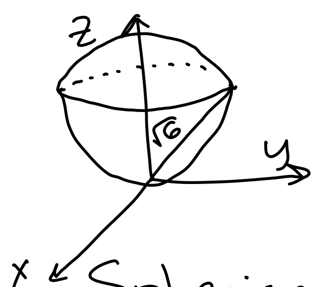
$z = \sqrt{1-x^2}$

$x = \sqrt{1-z^2}$

$$V = \int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{x} dy\,dz\,dx + \int_0^1\int_{\sqrt{1-x^2}}^{1}\int_0^{\sqrt{1-z^2}} dy\,dz\,dx$$

$$V = \int_0^1\int_0^{\sqrt{1-z^2}}\int_0^{x} dy\,dx\,dz + \int_0^1\int_{\sqrt{1-z^2}}^{1}\int_0^{\sqrt{1-z^2}} dy\,dx\,dz$$

③

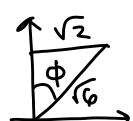


Cylindrical Coordinates

$$V = \int_0^{2\pi}\int_0^{\sqrt{2}}\int_{r^2}^{\sqrt{6-r^2}} r\,dz\,dr\,d\theta$$

Spherical Coordinates

recall: $r = \rho\sin\phi$



$\phi = \arcsin\frac{\sqrt{2}}{\sqrt{6}}$

$$V = \int_0^{2\pi}\int_0^{\arcsin\frac{1}{\sqrt{3}}}\int_0^{\sqrt{6}} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$+ \int_0^{2\pi}\int_{\arcsin\frac{1}{\sqrt{3}}}^{\frac{\pi}{2}}\int_0^{\cot\phi\csc\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$z = x^2+y^2 = r^2$

$\cancel{\rho}\cos\phi = \rho^{\cancel{2}}\sin^2\phi$

$\rho = \cos\phi / \sin^2\phi = \cot\phi\csc\phi$