Graphing Surfaces and Contour Maps

- **To graph *z* = *f* (*x*, *y*), use Plot3D. You have to specify a domain for *x* and for *y*. Do not type the "*z* =" part.**

```
Plot3D[x^2 + y^2, {x, -3, 3}, {y, -3, 3}]
```

- **You can use options such as PlotRange, BoxRatios, PlotStyle, etc. Try adding each one individually while evaluating to see what it does.**

```
Plot3D[x^2 + y^2, {x, -3, 3}, {y, -3, 3}, PlotRange → {{-5, 5}, {-5, 5}, {0, 20}},
 BoxRatios → Automatic, PlotStyle → Green, Mesh → False, Boxed → False,
 Axes → True, AxesEdge → {-1, -1}, AxesOrigin → {0, 0, 0}]
```

- **ContourPlot is similar to Plot3D in how you set it up. Tooltips appear when you point to a contour line (level curve).**

```
ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3}]
```

```
ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3}, ContourLabels → All]
```

```
ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3}, ContourLabels → Automatic, Contours → 100]
```

- **Try the options listed below to figure out what each does.**
  (Note: Listing Contours like below shows the contour line for z = each of those values.)

```
ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3},
 Frame → False, Axes → True, ContourShading → None, ContourStyle → Red,
 ContourLabels → Automatic, Contours → {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}]
```

- **Below are different options for the same function. (Note: This specifies how many contour lines you want to see.)**

```
ContourPlot[x^2 + y^2, {x, -3, 3}, {y, -3, 3},
 ColorFunction → "Rainbow", ContourLines → False, Contours → 50]
```

For other ColorFunction inputs, delete "Rainbow" and put your cursor after the →. Then go to the menu Palettes >> ColorSchemes. Choose one you like and click "Insert."

- **Use what you have learned to graph the following surfaces and their contour maps. (Remember to use appropriate *Mathematica* syntax such as Cos[x] or x^2.)**

$z = \cos(x)\sin(y)\,, \quad 0 \le x \le 4\pi\,, \quad 0 \le y \le 4\pi$

$z = -\dfrac{4x}{x^2 + y^2 + 1}\,, \quad -3 \le x \le 3\,, \quad -3 \le y \le 3$

$z = \cos\left(\dfrac{1}{4}\left(x^2 + 2y^2\right)\right), \quad -3 \le x \le 3\,, \quad -3 \le y \le 3$

$z = y^2 - x^2\,, \quad -3 \le x \le 3\,, \quad -3 \le y \le 3$

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4 Dimensions

- **To graph *w* = *f* (*x*, *y*, *z*) in a 3D space, you must use ContourPlot3D. See the example below.**

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}]
```

- **Try it with the options below.**
  (Note: If it is too slow, use fewer contours and do not use Opacity.)

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → 5,
 Mesh → False, Boxed → False, Axes → False, ContourStyle → {{Purple, Opacity[0.4]}}]
```

- **Try changing ContourStyle and adding ColorFunction as below.**

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → 5, Mesh → False,
 Boxed → False, Axes → False, ContourStyle → Opacity[0.4], ColorFunction → "Rainbow"]
```

- **Choose one specific contour (level surface) to show. Start with -1 as below, then change to 0, then 1.**

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → {-1}]
```

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → {0}]
```

```
ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → {1}]
```

- **Manipulate!**

```
Manipulate[
 ContourPlot3D[x^2 + y^2 - z^2, {x, -4, 4}, {y, -4, 4}, {z, -4, 4}, Contours → {c}],
 {c, -2, 2}]
```

- **Try these other functions with ContourPlot3D**
  (Note: Do NOT go crazy with your functions. ContourPlot3D already takes a lot of time to compute. Avoid using trig functions.)

$f(x, y, z) = x^2 + y^2 + z^2$, $0 \le x \le 4$, $-4 \le y \le 4$, $-4 \le z \le 4$ Use the option BoxRatios−>Automatic

$f(x, y, z) = x + y + z$, $-4 \le x \le 4$, $-4 \le y \le 4$, $-4 \le z \le 4$ Use the options Contours−>10 and ColorFunction−>"Rainbow"

$f(x, y, z) = x\,y\,z$, $-4 \le x \le 4$, $-4 \le y \le 4$, $-4 \le z \le 4$
Use the options Contours−>{−4, 0, 4} and ColorFunction−>"Rainbow" What is going on with this function and graph?

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