RegionPlot and RegionPlot3D for Multiple Integrals

RegionPlot

$$\int_0^4 \int_{\sqrt{x}}^2 f(x, y)\, dy\, dx$$

- **Plot the domain of the integral above using the code below.**
  Note: Always list the x-domain (horizontal) first then the y-domain (vertical) second.

```
RegionPlot[Sqrt[x] < y < 2, {x, 0, 4}, {y, 0, 2}]
```

- **Insert the following options, as shown. Note what each option does.**

```
RegionPlot[Sqrt[x] < y < 2, {x, 0, 4}, {y, 0, 2}, Axes → True,
 AxesLabel → {"x", "y"}, Frame → False, AspectRatio → Automatic]
```

- **To see more options, evaluate the code below.**

```
Options[RegionPlot]
```

$$\int_0^1 \int_{y^2}^{\sqrt[3]{y}} f(x, y)\, dx\, dy$$

- **In the margin, sketch what you predict this integral domain will look like.**

```
region = RegionPlot[y^2 < x < y^(1/3), {x, -1, 2}, {y, -1, 2},
   Axes → True, AxesLabel → {"x", "y"}, Frame → False,
   AspectRatio → Automatic, BoundaryStyle → Red, PlotStyle → Yellow]

Show[region,
 Plot[x^3, {x, -1, 2}],
 ParametricPlot[{t^2, t}, {t, -1, 2}]
]
```

$$\int_0^2 \int_x^{2x} f(x, y)\, dy\, dx$$

- **In the margin, sketch what you predict this integral domain will look like.**

```
region = RegionPlot[(x < y < 2 x) && (x < 2), {x, 0, 3}, {y, 0, 5},
   Axes → True, AxesLabel → {"x", "y"}, Frame → False, AspectRatio → Automatic]

Show[region,
 Plot[{x, 2 x}, {x, 0, 3}],
 ParametricPlot[{2, t}, {t, 0, 5}]
]
```

$$\int_0^3 \int_{\frac{4y}{3}}^{\sqrt{25-y^2}} f(x, y)\, dx\, dy$$

- **Sketch your prediction and then plot the domain using *Mathematica*.**

RegionPlot3D

Note: Delete 3D graphics before saving your file.

- **Try the plot below, then add the various options shown in the next line of code. Note the effect of each option. (The inequalities are structured differently in each of the below examples, but it can be the same since either way works.)**

```
RegionPlot3D[ (x / 3 < z < Sin[x] Cos[y] + 5) && (0 < x < 2 Pi) && (0 < y < 2 Pi),
 {x, -1, 7}, {y, -1, 7}, {z, 0, 6}]
```

```
RegionPlot3D[ (z > x / 3) && (z < Sin[x] Cos[y] + 5) && (0 < x < 2 Pi) && (0 < y < 2 Pi),
 {x, -1, 7}, {y, -1, 7}, {z, 0, 6}, BoxRatios → Automatic, AxesEdge → {-1, -1},
 Boxed → False, AxesLabel → {"x", "y", "z"}, Mesh → False, PlotStyle → Purple]
```

- **Insert the option `PlotPoints->20` or `PlotPoints->40` and see what happens. Do NOT make this value too large.**
- **Fill in the limits of integration based on the above plot :**

$$\int \int \int f(x, y, z)\, dz\, dy\, dx$$

Plot the region for : $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$

Sketch the 3D region bounded below by $z = x^2 + y^2$ and above by $x^2 + y^2 + z^2 = 25$.

- **Plot the region described above using *Mathematica*. Hint: Use `{x,-3,3},{y,-3,3},{z,0,6}` for the domain of the plot.**