Chapter 7 – Brief Notes

$A\vec{x} = \lambda\vec{x}$ λ = eigenvalue, $\vec{x}$ = eigenvector corresponding to λ.

$\lambda\vec{x} - A\vec{x} = \vec{0}$

$(\lambda I - A)\vec{x} = \vec{0}$ ← Want this to have nontrivial solutions

⇒ Want $(\lambda I - A)$ to be singular

$\det(\lambda I - A) = 0$ ←

← Characteristic equation

↳ solve for λ.

For each λ, substitute into $(\lambda I - A)\vec{x} = \vec{0}$ and solve for $\vec{x}$.

To find a basis for the <u>eigenspace</u>, think of this as finding a basis for the <u>nullspace of $(\lambda I - A)$</u>.

Note: A is invertible iff $\lambda = 0$ is <u>not</u> an eigenvalue of A.

Note: Eigenvectors corresponding to distinct eigenvalues are linearly independent.

<u>Diagonalization Problem</u>: Given A ($n \times n$) find invertible P such that $P^{-1}AP$ is a diagonal matrix.

1. Find n linearly independent eigenvectors ← "basis"
   (Eigenspace basis vectors are already linearly independent as are those corresponding to different eigenvalues.)
2. P has the eigenvectors as its columns.
3. $P^{-1}AP = D$ where D is diagonal with the λ's as its entries in the order matching the columns of P.

Algebraic multiplicity (root of characteristic equation)

Geometric multiplicity (dimension of eigenspace)

For each λ, Geom. Mult. $\leq$ Alg. Mult.

A is diagonalizable iff Geom. Mult. = Alg. Mult. $\forall\lambda$.

<u>Orthogonal Diagonalization Problem</u>: Same, but now P is orthogonal (columns are orthonormal)

A must be symmetric.

When symmetric, eigenvectors from different eigenspaces are orthogonal.

Same steps, but now Gram-Schmidt each eigenspace basis.

Abby Brown 5/2005