TPHS Linear Algebra — SDSU Math 254 Name: ____________________

Quiz #2 (1.6 - 2.3) _____ Per.: _____

Scientific calculator allowed. 20 pts.

Clearly show *all* work. Simplify and circle your answers.

1) a) For $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$ find A^{-1}. (3 pts.)

Please label your steps as you go.

Hint: If you find a common multiple for the first column, you can do this without fractions.

b) Use an inverse matrix to solve the following system that is in $A\mathbf{x} = \mathbf{b}$ form.

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= -1 \\ 3x_1 + 3x_2 + x_3 &= 1 \\ 2x_1 + 4x_2 + x_3 &= -2 \end{aligned}$$

(3 pts.)

2) Let A and B be symmetric $n \times n$ matrices. Use the definition of a symmetric matrix to prove that if AB is a symmetric matrix, then A and B commute. (3 pts.)

3) What is the name for a matrix that is both upper triangular and lower triangular? (1 pt.)

For #4 and 5 be sure to clearly show your work and/or explain how you arrived at your answer.

4) (2 pts.) Find $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{vmatrix}$.

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5) (2 pts.) Find $\begin{vmatrix} 5 & 8 & -4 & 2 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{vmatrix}$.

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6) *True or False?* If the determinant of an $n \times n$ matrix A is nonzero, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. (2 pts.)

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7) *True or False?* If A is an invertible $n \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions. (2 pts.)

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8) Find the eigenvalues of $A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$. I.e., find λ in the equation $\det(\lambda I - A) = 0$. (2 pts.)

Bonus) An invertible square matrix A is called *orthogonal* if $A^{-1} = A^T$. Prove that if A is an orthogonal matrix, then $\det(A) = \pm 1$. (+2 pts.)