Name: ___________________________ Per. ____

Illustrating the Definitions of Limits

Part I – Limit Definitions with *Mathematica*.

- Open the corresponding *Mathematica* notebook file.
- Open the section for $\lim_{x \to c} f(x) = L$ (Finite c, Finite L) by clicking the down arrow on the left.
- Click on the point P (red point) on the curve and drag it to the coordinates (4.00, 3.00). Hold the "Alt" key while dragging to increase accuracy. Ctrl-Alt together while dragging will slow it further.
- Click on the line "L + Epsilon" (green point) and drag it up and down. Move it until $\varepsilon = 1.00$.
- Move the vertical line for "Delta 1" (left orange point) until it matches the intersection of "L - Epsilon" and the curve. Do the same thing for the other vertical delta line.
- You can adjust the magnification with the lower slider to zoom in for more accuracy, as needed.

- Fill in the blanks for this problem:

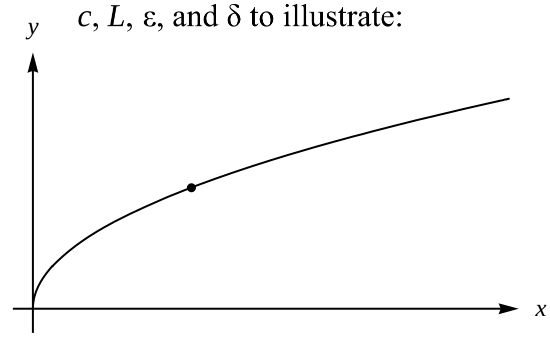
  For $\lim_{x \to 4} \frac{3\sqrt{x}}{2} = 3$, if $\varepsilon = 1.00$, then δ_1 (Delta 1) = ________ and δ_2 (Delta 2) = ________.

  Therefore, $\left|\frac{3\sqrt{x}}{2} - 3\right| < 1.00$ whenever $|x - 4| <$ ________ (δ). For δ, choose a single δ.

- Fill in the chart below:

c	L	ε	δ_1	δ_2	δ
4	3	1.00			
4	3	0.50			
4	3	0.10			
2		0.35			
2		0.05			

Complete the sketch with c, L, ε, and δ to illustrate:



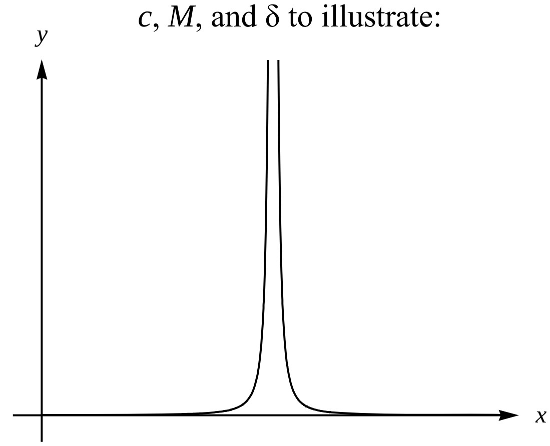
- Open the section for $\lim_{x \to c} f(x) = \infty$ (Finite c, Infinite L).
- Fill in the blanks for this problem: (If you can't get M to be exactly 40, get it as close as you can.)

  For $\lim_{x \to 4} \frac{1}{(x-4)^2} = \infty$, if $M =$ ___40___, then let $\delta =$ _________.

- Therefore, $\frac{1}{(x-4)^2} > 40$ whenever $|x - 4| <$ ________ (δ).
- Fill in the chart below:

c	L	M	δ
4	$f(x) \to \infty$	40	
4	$f(x) \to \infty$	70	
4	$f(x) \to \infty$		0.10

Complete the sketch with c, M, and δ to illustrate:



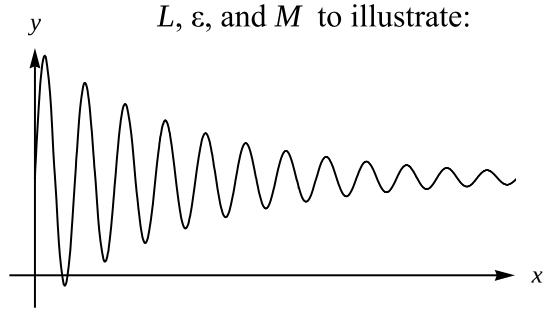
– Continued –

- Open the section for $\lim_{x\to\infty} f(x) = L$ (Infinite c, Finite L).
- Fill in the blanks for this problem:
  For $\lim_{x\to\infty} 4(\sin 5x)e^{-x/5} + 3 = 3$, if ε = 2.00, then let M = ________.
- Therefore, $\left|\left(4(\sin 5x)e^{-x/5} + 3\right) - 3\right| < 2.00$ whenever $x >$ ________ (M).
- Fill in the chart below:

c	L	ε	M
$x \to \infty$	3	2.00	
$x \to \infty$	3	1.00	
$x \to \infty$	3	0.50	
$x \to \infty$	3	0.10	

Note: You can scroll right to see the graph for larger x.

Complete the sketch with L, ε, and M to illustrate:

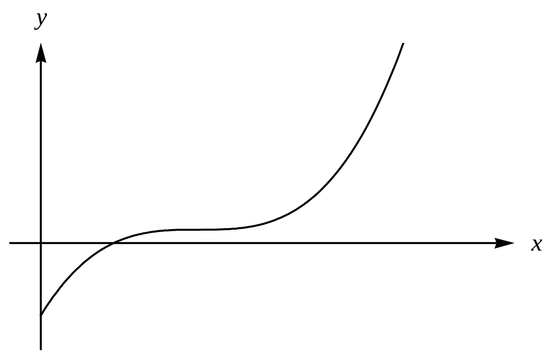


- Open the section for $\lim_{x\to\infty} f(x) = \infty$ (Infinite c, Infinite L).
- Fill in the blanks for this problem:
  For $\lim_{x\to\infty} (x-4)^3 + 10 = \infty$, if M = 60, then let N = ________.
- Therefore, $(x-4)^3 + 10 > 60$ whenever $x >$ ________ (N).
- Fill in the chart below:

c	L	M	N
$x \to \infty$	$f(x) \to \infty$	60	
$x \to \infty$	$f(x) \to \infty$	160	
$x \to \infty$	$f(x) \to \infty$	260	
$x \to \infty$	$f(x) \to \infty$	1000	

You can adjust the plot range to see larger y values.

Complete the sketch with M and N to illustrate:



Part II – Observations

What did you learn from doing this activity? For which situations do you use ε? For which situations do you use δ? How do we deal with infinity? What else did you notice?