Students are expected to complete homework assignments on their own before referring to the following pages. The answers and hints are designed to check work and clarify problems. The original intent of the layout was for display in class after assignments had been completed. Students should use the following information as help to understand the exercises and master the concepts.

# Calculus D

# Chapter 13

## Even Answers & Hints for Homework

# 13.1 Even Answers

(2) $\vec{r}(t) = \frac{t-2}{t+2}\hat{\imath} + \sin t\,\hat{\jmath} + \ln(9-t^2)\hat{k}$

$\hookrightarrow t \neq -2$ $\qquad\qquad$ $\hookrightarrow 9-t^2 > 0$

Domain: $(-3,-2) \cup (-2,3)$ $\qquad\qquad$ $-3 < t < 3$

(4) $\lim\limits_{t \to 0} \left\langle \frac{e^t - 1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{1+t} \right\rangle = \left\langle 1, \frac{1}{2}, 3 \right\rangle$

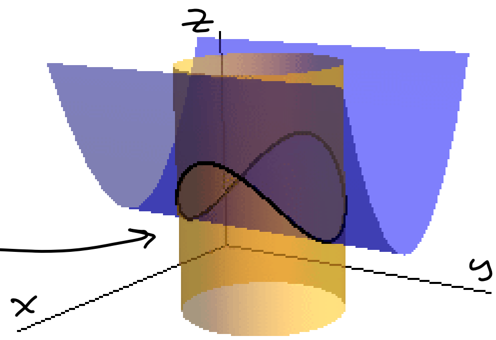
(19) VI (20) II (21) IV (22) I (23) V (24) III

(26) $z = x^2 \leftarrow$ Surface?

$x^2+y^2=1 \leftarrow$ Surface?

Plug parametric equations into

$\vec{r}(t) = \langle \sin t, \cos t, \sin^2 t \rangle$




(36) $x^2+y^2=4$, $z=xy$

Use trig: $x = 2\cos t$, $y = 2\sin t$

$z = 2\cos t\, 2\sin t = 2\sin 2t$

$\vec{r}(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath} + 2\sin 2t\,\hat{k}$

(38) $z = 4x^2 + y^2$, $y = x^2$

Let $x = t \Rightarrow y = t^2$ and $z = 4t^2 + (t^2)^2$

$\vec{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath} + (4t^2+t^4)\hat{k}$

(37) Book answer uses $x=t$ to avoid $\pm\sqrt{\phantom{x}}$ in answer.

my answer: $y = t \Rightarrow z = 1+t$

and $x = \pm\sqrt{z^2-y^2} = \pm\sqrt{(1+t)^2-t^2}$

$\vec{r}(t) = \pm\sqrt{(1+t)^2-t^2}\,\hat{\imath} + t\,\hat{\jmath} + (1+t)\hat{k}$

# 13.2 + 13.4 Even Answers

13.2

18 $\vec{T}(1) = \frac{2}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k}$

20 $\vec{T}(\frac{\pi}{4}) = \frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$

24 $x = t+1,\ y = t,\ z = t$

26 $x = t,\ y = t+2,\ z = 2t+1$

28 $x = -t+\sqrt{3},\ y = \sqrt{3}t+1,\ z = -4\sqrt{3}t + 2$

13.4

18a $\vec{r}(t) = \frac{1}{6}t^3\hat{\imath} + (e^{t} - t)\hat{\jmath} + (e^{-t} + 2t)\hat{k}$

# 13.3 and 13.4 Even Answers

## 13.3

44. $\vec{T}(t) = \langle -\sin t \cos t, \cos^2 t, -\sin t \rangle$

$\vec{T}(0) = \langle 0, 1, 0 \rangle$

$\vec{N}(0) = \langle -\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \rangle$

## 13.4

34. $a_T = \dfrac{4t-4}{\sqrt{4t^2-8t+5}}$, $a_N = \dfrac{2}{\sqrt{4t^2-8t+5}}$

36. $a_T = \dfrac{4t}{\sqrt{4t^2+10}}$, $a_N = \dfrac{2\sqrt{10}}{\sqrt{4t^2+10}}$

38. $a_T = \dfrac{4\sin 2t \cos 2t}{\sqrt{1+2\sin^2 2t}}$, $a_N = \dfrac{2\sqrt{2}\,|\cos 2t|}{\sqrt{1+2\sin^2 2t}}$

Part II

# 13.3 Even Answers

(14) $\vec{r}(s) = \left(\frac{s}{\sqrt{2}}+1\right)\cos\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\hat{\imath}$
$+ 2\hat{\jmath}$
$+ \left(\frac{s}{\sqrt{2}}+1\right)\sin\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\hat{k}$

$t = \frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)$

(19) **Also see #19 on the next page.**
**The following note goes with #19:**

*There is a hint on the homework assignment sheet, but the algebra is certainly more challenging on this one. The book also uses some simplifying (such as multiplying by e^t/e^t) that I wouldn't expect you to do. So, I worked out the problem without that type of simplification and then showed how it links to the answers in the book. See attached file.*

*Please don't overly stress about completely getting through this problem. The algebra won't be this complicated on the test either. Think of this one as a fun challenge. I certainly did!*

13.3 #19 $\vec{r}(t) = \langle \sqrt{2}\,t, e^t, e^{-t} \rangle$ Find $\vec{T}(t)$ and $\vec{N}(t)$.
(and curvature $K(t)$)

$$\vec{r}\,'(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{(\sqrt{2})^2 + (e^t)^2 + (-e^{-t})^2}$$

$$= \sqrt{2 + e^{2t} + e^{-2t}}$$

$$e^{2t} + 2 + e^{-2t}$$
$$(e^t)^2 + 2e^te^{-t} + (e^{-t})^2$$
$$(e^t + e^{-t})^2$$

$$= \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$$

$$\boxed{\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} = \frac{\langle \sqrt{2}, e^t, -e^{-t} \rangle}{e^t + e^{-t}}} \leftarrow \text{OK}$$

B.O.B. simplifies like this: $\dfrac{\langle \sqrt{2}, e^t, -e^{-t} \rangle}{e^t + e^{-t}} \cdot \dfrac{e^t}{e^t}$ ← Multiplying by 1

$$\boxed{\vec{T}(t) = \frac{1}{e^{2t}+1} \langle \sqrt{2}e^t, e^{2t}, -1 \rangle} \leftarrow \text{B.O.B. Answer}$$

Let's see how it works out if I use my answer for **T**(t) rather than book's simplified answer.

Using "OK" answer: $\vec{T}(t) = \left\langle \sqrt{2}(e^t + e^{-t})^{-1}, \dfrac{e^t}{e^t + e^{-t}}, \dfrac{-e^{-t}}{e^t + e^{-t}} \right\rangle$

$$\vec{T}\,'(t) = \left\langle \frac{-\sqrt{2}(e^t - e^{-t})}{(e^t + e^{-t})^2}, \frac{(e^t + e^{-t})e^t - e^t(e^t - e^{-t})}{(e^t + e^{-t})^2}, \frac{(e^t + e^{-t})e^{-t} + e^{-t}(e^t - e^{-t})}{(e^t + e^{-t})^2} \right\rangle$$

$$\vec{T}\,'(t) = \frac{1}{(e^t + e^{-t})^2} \langle -\sqrt{2}(e^t - e^{-t}), 2, 2 \rangle$$

$$\|\vec{T}\,'(t)\| = \frac{1}{(e^t + e^{-t})^2} \sqrt{2(e^{2t} - 2 + e^{-2t}) + 4 + 4}$$

$$2e^{2t} - 4 + 2e^{-2t} + 8$$
$$= 2e^{2t} + 4 + 2e^{-2t}$$
$$= 2(e^{2t} + 2 + e^{-2t})$$
$$= 2(e^t + e^{-t})^2$$

$$= \frac{1}{(e^t + e^{-t})^2} \sqrt{2}(e^t + e^{-t}) = \frac{\sqrt{2}}{e^t + e^{-t}}$$

Reminder: $\|k\vec{v}\| = |k|\,\|\vec{v}\|$

$$\vec{N}(t) = \frac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|} = \frac{1}{(e^t + e^{-t})^2} \langle -\sqrt{2}(e^t - e^{-t}), 2, 2 \rangle \cdot \frac{(e^t + e^{-t})}{\sqrt{2}}$$

← mult. by reciprocal

$$\boxed{\vec{N}(t) = \frac{1}{e^t + e^{-t}} \langle e^{-t} - e^t, \sqrt{2}, \sqrt{2} \rangle} \leftarrow \text{OK}$$

B.O.B. Simplification: Multiply above by $\dfrac{e^t}{e^t}$

$$\boxed{\vec{N}(t) = \frac{1}{e^{2t}+1} \langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \rangle} \leftarrow \text{B.O.B. Answer}$$

mult. by reciprocal

$$K(t) = \frac{\|\vec{T}\,'(t)\|}{\|\vec{r}\,'(t)\|} = \frac{\sqrt{2}}{(e^t + e^{-t})} \cdot \frac{1}{(e^t + e^{-t})} \Rightarrow \boxed{K(t) = \frac{\sqrt{2}}{(e^t + e^{-t})^2}} \leftarrow \text{OK}$$

Multiply by $\dfrac{e^{2t}}{e^{2t}}$ for B.O.B. answer. $\boxed{K(t) = \dfrac{\sqrt{2}e^{2t}}{(e^{2t}+1)^2}}$ ← B.O.B.