How to do a Cross Product

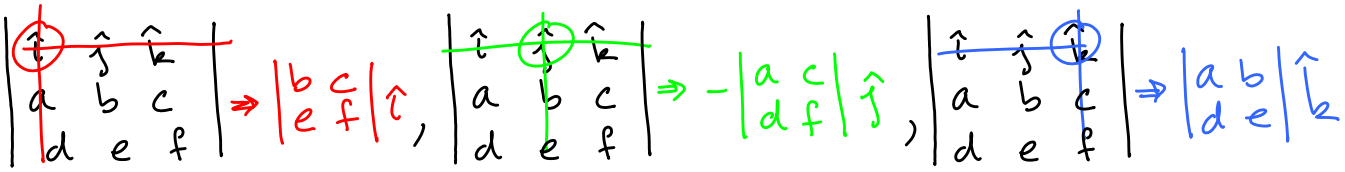
A cross product is a 3×3 determinant. You did determinants in Algebra II and/or Precalculus.

Recall a 2×2 determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

You may have been taught "gamma" or "fish."

For a cross product, the determinant is expanded across the first row. For each of $\hat{i}$, $\hat{j}$, and $\hat{k}$, imagine crossing out its row and column. The four entries left are a 2×2 determinant that gets multiplied by $\hat{i}$, $\hat{j}$, or $\hat{k}$.

Important Note: The $\hat{j}$ term is multiplied by a negative.



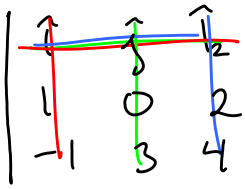
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ d & e & f \end{vmatrix} = \begin{vmatrix} b & c \\ e & f \end{vmatrix}\hat{i} - \begin{vmatrix} a & c \\ d & f \end{vmatrix}\hat{j} + \begin{vmatrix} a & b \\ d & e \end{vmatrix}\hat{k}$$

$$= (bf - ce)\hat{i} - (af - cd)\hat{j} + (ae - bd)\hat{k}$$

↑ Do NOT memorize this formula. Go through the process each time.

Note: The bars on the sides are important notation. Without the bars, it does not mean the same thing. Also, the $\hat{i}$, $\hat{j}$, $\hat{k}$ must be in the top row.

Example: $\langle 1,0,2 \rangle \times \langle -1,3,4 \rangle$



$$= (0-6)\hat{i} - (4-{-2})\hat{j} + (3-0)\hat{k}$$

$$= -6\hat{i} - 6\hat{j} + 3\hat{k}$$

or $= \langle -6, -6, 3 \rangle$

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